GRADE 3 • MODULE 1

Properties of Multiplication and Division and Solving Problems with Units of 2–5 and 10

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Terminology

Array (a set of numbers or objects that follow a specific pattern, a matrix)
Column (e.g., in an array)
Commutative Property/Commutative (e.g., rotate a rectangular array 90 degrees to demonstrate that factors in a multiplication sentence can switch places)
Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
Equation (a statement that 2 expressions are equal. E.g., 3 × 4 = 12)
Distribute (with reference to the Distributive Property; e.g. In 12 × 3 = (10 × 3) + (2 × 3) the 3 is multiplier for each part of the decomposition)
Divide/division (partitioning a total into equal groups to show how many equal groups add up to a specific number. E.g., 15 ÷ 5 = 3)
Fact (used to refer to multiplication facts, e.g., 3 × 2)
Factors (i.e., numbers that are multiplied to obtain a product)
Multiplication/multiply (an operation showing how many times a number is added to itself e.g., 5 × 3 =15)
Number of groups (factor in a multiplication problem that refers to the total equal groups)
Parentheses (e.g., ( ) used around a fact or numbers within an equation)
Quotient (the answer when one number is divided by another)
Rotate (turn, used with reference to turning arrays 90 degrees)
Row/column (in reference to rectangular arrays)
Size of groups (factor in a multiplication problem that refers to how many in a group)
Unit (i.e., one segment of a partitioned tape diagram)
Unknown (i.e., the "missing" factor or quantity in multiplication or division)

Topic A: Multiplication and the Meaning of the Factors

Lesson 1: Understand "equal groups of" as multiplication

- skip count to find the total number of objects

2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 20
10 twos
10 groups of two is 20.

- use repeated addition to show equal groups

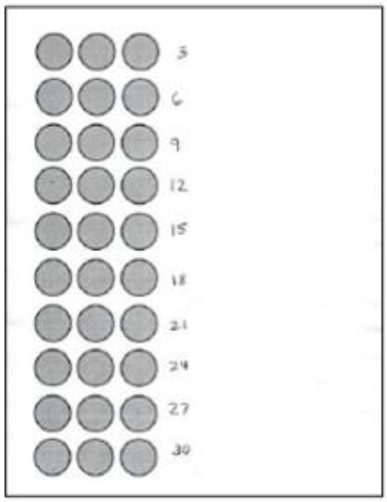
2 + 2 + 2 + 2 + 2 + 2 = 12
6 twos = 12
6 x 2 = 12

- write multiplication sentences from equal groups

3 x 4 = 12

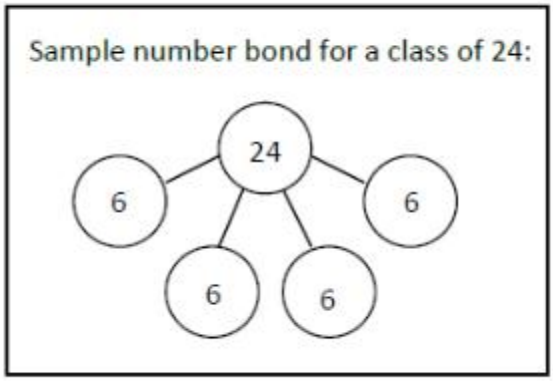
Lesson 2: Relate multiplication to the array model.

- If you skip-count 10 rows of 3 circles each and the total is 30. It just means 10 groups of 3 and when you add 10 threes, you get 30! However, writing 10 × 3 is a lot easier than writing out 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3



Lesson 3: Interpret the meaning of factors- the size of the group or the number of groups

- Given a multiplication sentence, draw an array. Now draw a number bond, placing the total in a circle and parts coming from the total. Write the factor representing the size of each group, which represents the parts.



Topic B: Division as an Unknown Factor Problem

Lessons 4 and 5: Understand the meaning of the unknown as the size of the group in division

- (Concrete to Abstract) Think of division as fair sharing.

  Yesterday out neighbor bought a new pack of 18 markers. He wanted to share them with me, so this morning he divided them into 2 equal groups. Now I have a bunch of new markers for making out charts! Do you want to know how many he gave me?

  First, decide that you are trying to find the size of the group, not the number of groups (the number of groups is 2, as given in the problem). Take 18 counters, which represent the markers, and divide into two equal groups 9fair share). If you cont how many are in each group, you get 9! So, 18 ÷ 2 = 9!

- (Pictorial to Abstract) Look at a picture to help you write a division equation

The 12 represents the total number of stars, the 3 represents the number of equal groups and the 4 represents the size of each group. Therefore the equation, is 12 ÷ 3 = 4!

- (Abstract to Pictorial) Analyze the equation for the meaning of the solution

  8 ÷ 4 = ___. If 8 is the total and 4 is the number of groups, then the unknown factor must represent the size of the groups!

Lesson 6: Interpret the unknown in division using the array model.

- Relate division to an array model

This array shows a total of 20, that has 4 rows, with 5 dots in each row.

- Use an array to relate the unknown factor in multiplication to the quotient in division

This array shows the division sentence 15 ÷ 3 = 5, where the quotient represents the size of the groups

- Relate multiplication and division.

  ___ x 3 = 24

Skip count and track the number of threes to solve

3, 6, 9, 12, 15, 18, 21, 24

So, 8 x 3 = 24

A related division sentence where the quotient represents the unknown factor would be

24 ÷ 3 = 8

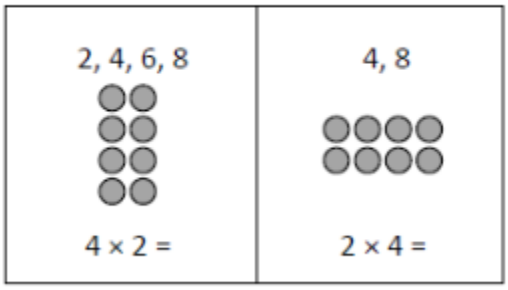
So, 24 divided into threes makes 8 groups. The unknown factor represents the same as the quotient.

Topic C: Analyze Arrays to Multiply Using Units of 2 and 3

Lessons 7 and 8: Demonstrate the Commutativity of multiplication and practice related facts by skip counting objects in array models.

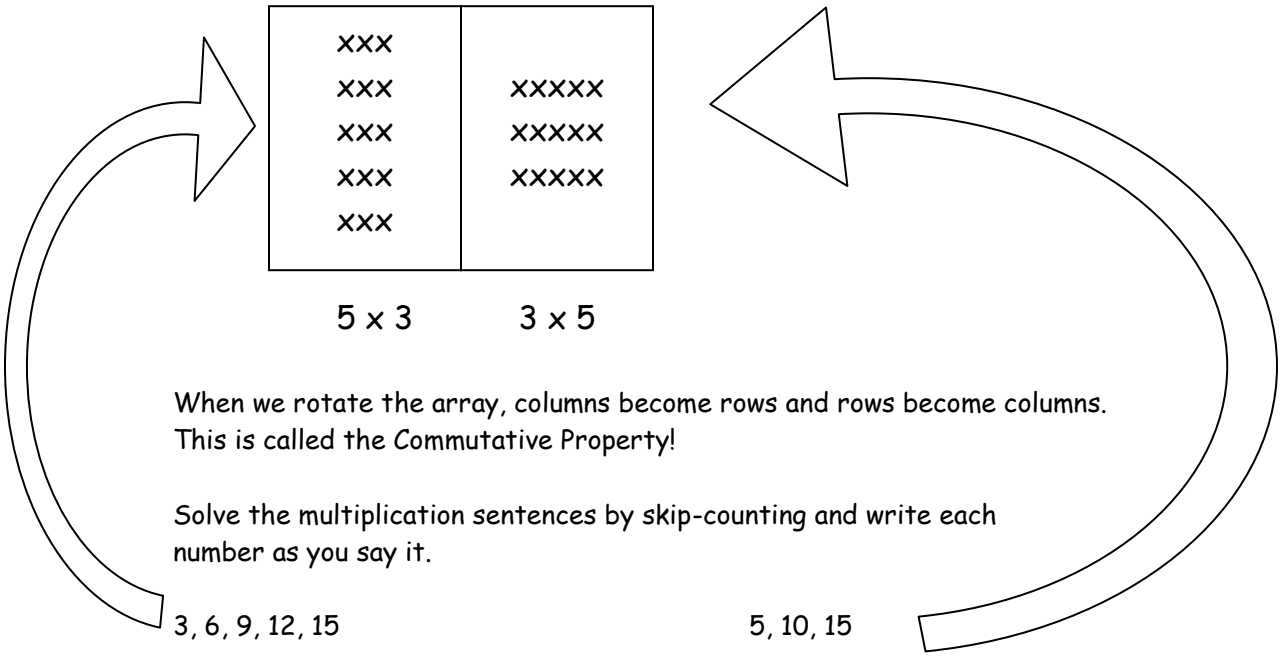
Rotate 90 degrees- move whiteboard from horizontal to vertical
Columns- the vertical arrangement of objects in an array
Row- the horizontal arrangement of objects in an array
Commutative- changing the order of factors in multiplication doesn't change the total

- Rotate arrays 90 degrees to see the groups and rows in different ways



- Interpret rows and columns in rotated arrays

  Draw an array that shows 5 rows and 3 columns, then rotate to show 3 rows and 5 columns



When we rotate the array, columns become rows and rows become columns. This is called the Commutative Property!

Solve the multiplication sentences by skip-counting and write each number as you say it.

3, 6, 9, 12, 15 5, 10, 15

Lesson 9: Find related multiplication facts by adding and equal groups in array models

- Add 2 known smaller facts to solve an unknown larg

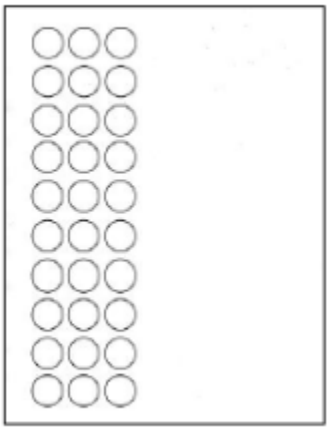
  7 x 3

  First, using a threes array template, cover part of the array so that 5 rows of 3 is uncovered and **5x3=15**

  Then, move the paper so it shows 7x3

  Now, shade the two rows you added and **2x3=6**

  There are 5 threes in 5x3 and you added 2 threes to make the array show 7x3, so 5 threes + 2 threes gives me 7 threes



  Therefore,

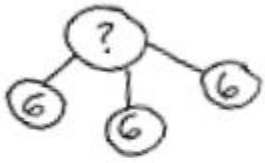
7 threes = 5 threes + 2 threes

7 x 3 = 5 x 3 + 2 x 3

21 = 15 + 6

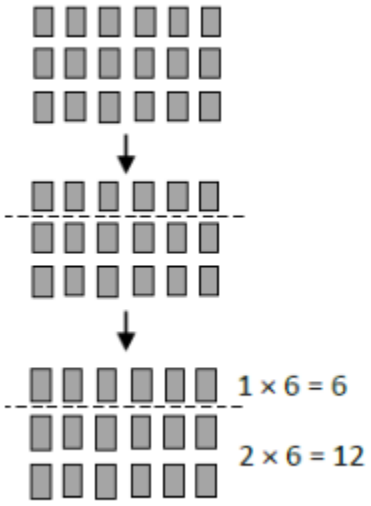
Lesson 10: Model the distributive property with arrays to decompose units as a strategy to multiply

A guitar has 6 strings. How many strings are there on 3 guitars?
Write a multiplication sentence to solve.



$3 \times 6 = 18$
There are 18 strings on 3 guitars.

First, draw an array to represent the total number of guitar strings. Let the number of strings on a guitar be on one row. **Then**, make a dotted line below the first row to show just 1 guitar. **Now**, write and solve a multiplication sentence to describe each part of your array.



Therefore,

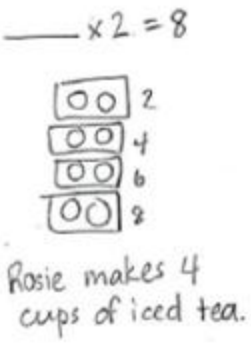
6 + 12 = 3 sixes
(1 × 6) + (2 × 6) = 3 sixes
(1 × 6) + (2 × 6) = 6 + 12

Topic D: Division Using Units of 2 and 3

Lesson 11: Model division as the unknown factor in multiplication using arrays and tape diagrams

- Relate arrays to tape diagrams, modeling division where the quotient represents the number of groups.

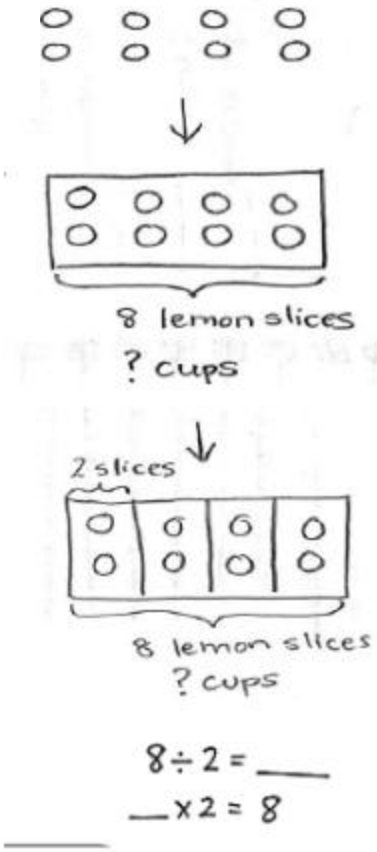
Rosie puts 2 lemon slices in each cup of iced tea. She uses a total of 8 slices. How many cups of iced tea does Rosie make?



The columns in this array show the number of lemon slices in 1 cup of Rosie's iced tea. The unknown is the number of cups or groups.

We can count the number of columns to find how many cups.

2 times 4 equals 8, so 8÷2=4

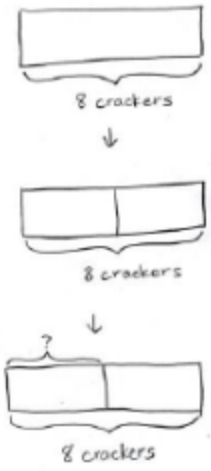


Lesson 12: Interpret the quotient as the number of groups or the number of objects in each group using units of 2.

- Model division where the unknown represents the number of objects in each group.

  2 students equally share 8 crackers. How many crackers does each student get?

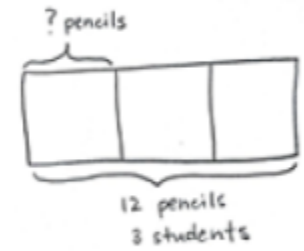
  First, draw a rectangle to represent the total, 8. **Now**, divide the picture into two equal parts. **Next**, decide what each part should be, keeping in mind that they have to be equal. Decide that each part must be 4. If you think 2 x __ =8, the missing or unknown number is 4!



Lesson 13: Interpret the quotient as the number of groups or the number of objects in each group using units of 3

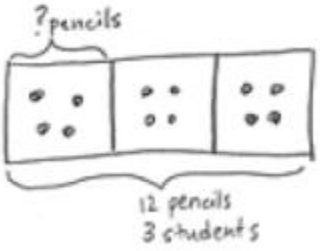
- Draw and analyze tape diagrams to determine the unknown

  3 students equally share a pack of 12 pencils.



We know that there are 12 total pencils and that it's divided into 3 parts (which represent number of students). We don't know how many pencils each student gets, that's the unknown, so 12÷3= ___.

If we put equal amounts of dots in each of the parts, we find that each student gets 4 pencils, so 12÷3=4!



Topic E: Multiplication and Division Using Units of 4

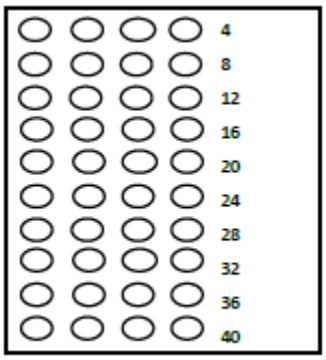
Lesson 14: Skip-Count objects in models to build fluency with multiplication facts using units of 4

- Skip-count by fours using an array to multiply

  Count to 4o using the array. Hum the number you count as you point to each dot. For the last dot in each row, say the number out loud and write it to the right of the row.

  Given a multiplication fact, give the answer.
  "What is 4x4?" **16**

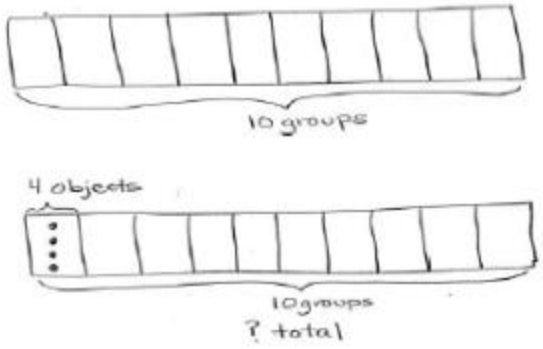
  Given the answer, indicate the equation.
  "The answer is 20, what is the equation?" **5x4**



- Use a tape diagram to model and solve multiplication

  Draw a tape diagram that represents the number of groups shown on the array template, **then** draw dots in each group. There are 4 dots in each group, **so** label one unit as 4 objects. **Now,** label the total as unknown and skip-count units to find the total value of your tape diagram.

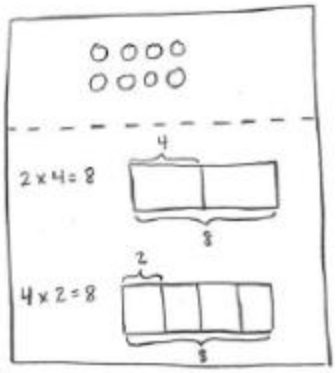
  4, 8, 12, 16, 20, 24, 28, 32, 36, 40, so 10x4=40



Lesson 15: Relate arrays to tape diagrams to model the commutative property of multiplication

- (Pictorial) Relate arrays with tape diagrams

  Draw an array with 2 rows and 4 columns on the top of a paper.
  Now, remembering the commutative property (rotate paper if necessary), write two multiplication equations for the array. Next to each equation, draw and label a tape diagram to match.

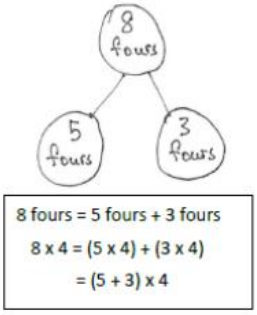


- (Pictorial-Abstract) Model commutativity using arrays and tape diagrams

  First draw arrays to match equations, then write equations for each array. Now draw and label tape diagrams to represent the commutativity for each set of facts.

Lesson 16: Use the distributive property as a strategy to find related multiplication facts

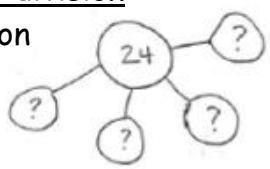
- We can break a larger fact into 2 smaller facts to help us solve it. Here we broke 8 fours into 5 fours and 3 fours to solve. So we can write an equation, 8 fours = 5 fours + 3 fours.
- (5+3) X 4 is another way of writing (5X4) + (3X4)



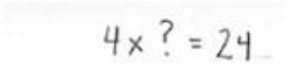
Lesson 17: Model the relationship between multiplication and division

- Use the number bond to relate multiplication and division

Using a number bond, find an unknown number.



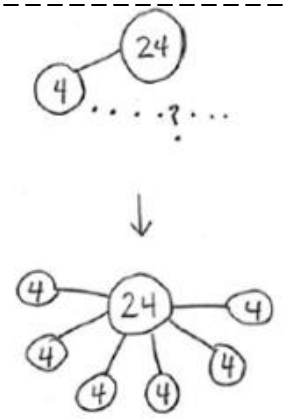
The number bond represents 24÷4, and if you split the total 4 ways, you get and answer of 6, so 24÷4=6.



The number bond shows 24÷4, but for this number bond you can skip-count by 4's to find the unknown factor. Each time you say a four, you can draw a new part of the number bond.

4, 8, 12, 16, 20, 24

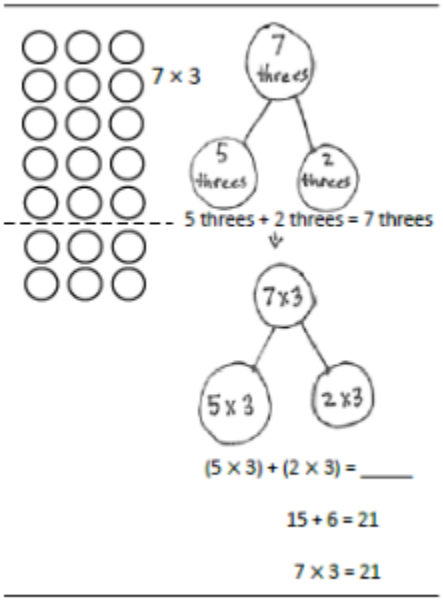
So, 6 fours make 24!



Topic F: Distributive Property and Problem Solving Using Units of 2-5 and 10

Lesson 18: Apply the distributive property to decompose units.

- Use number bonds to decompose numbers and apply the distributive property.



Lesson 19: Apply the distributive property to decompose units.

- Break apart and distribute using an array is a strategy for division